HOLOCENE SEA-LEVEL AND GEOID-DEFORMATION

HENRY JENSEN

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An often overlooked effect of a deglaciation is the change in the shape of the geoid. This is immediately followed up by the ocean-surface while the yielding of the sea floor happens with a certain delay. This means that eustatic curves determined from different stable areas do not necessarily coincide. The parameters commanding the mechanism are not very well known. However, a calculation carried out for Bermuda and New Zealand – using the Russian result that the Wisconsin ice age and the Würm ice age are not quite synchronous – shows that the difference between the eustatic curves from the two areas may be partly explained by the described effect.

Henry Jensen, Institut for geofysik, Haraldsgade 6, DK-2200 København N, Denmark. June 10th, 1971.

Holocene eustatic curves—that means graphs giving the global sea-level versus time for the last 20,000 years—have been calculated from various areas in the world. One might expect that the same curve would appear in all cases, and in fact the curves show the same trend—a sea-level rise at about 85 m—but they differ in details. Of course tectonic and isostatic movements may be insufficiently known, but nevertheless it seems as if the various curves differ more than expected even taking into account the possible errors in the levellings and in the age-determinations. Suggate (1968) writes that "differences in the sea-level curves of different regions may be real, and a search should be made for possible causes of such differences". The present paper presents one possible cause of the differences.

A peculiarity of some published curves is that at a certain time—viz. about 7500 years ago—they show a much better coincidence than before or after. In fact they seem to cross each other. This was pointed out by Suggate (1968) and confirmed by Mörner (1969) who called the phenomenon "the real Holocene eustatic sea level problem" (Mörner, 1971). Perhaps that feature also may partly be explained by the mechanism described below.

Effects of a deglaciation

When during a deglaciation a water-volume is induced into the ocean, the sea-level will rise in comparison with a stable landmass. This is a global effect. The mass of the melted ice divided by the area of the ocean will be called the equivalent elevation. At the first glance one could think that this would be the amount of water-rise observed.

However, other effects will occur. The removal of the ice mass will start an isostatic adjustment. This is often called glacio-isostasy and is a non-global effect. On the other hand, the water rising all over the globe will cause an elastic deformation of the solid Earth, and also an isostatic adjustment (hydro-isostasy), both on a global base.

The hydro-isostatic effect must be much smaller than the glacio-isostatic one, because the equivalent water height is much smaller than the ice thickness. The elastic effect is assumed to be approximately uniform over the globe, and will for that reason not be treated in the following.

The effects mentioned have been well investigated in the past by many authors (see e. g. Fairbridge, 1961). However, an additional effect has often been overlooked—the effect due to the deformation of the geoid.

The deformation of the geoid

Let us first assume that an ice-mass melts very rapidly. (This, of course, is an intellectual experiment, but in fact such events do happen. An example is the tapping of the Baltic Ice Lake on which occasion the water level fell 25 m in the course of half a year (Johansson, 1926).) Then the geoid of the new distribution of the masses of the Earth will be another than before. The subsequent change of the shape of the geoid will happen instantaneously. Because of the discrepancy between the new geoid and the mass-distribution in the solid Earth, the Earth will tend to adjust its shape. That is a rather slow process. Let us assume it to happen in such a way that the velocity in which an undulation of the geoid will be adjusted is proportional to the amplitude of the undulation itself. This means that we can speak about a relaxation time. Let us call it τ.

The shape of the deformation

When a certain mass is removed from a point of the surface of the Earth (the ice) and a similar mass (the water) is spread over the surface, it is easy to calculate the change in the surface gravity at a certain spherical distance Θ. The use of Stokes's formula (to be found in every textbook in higher geodesy, e. g. in Heiskanen & Moritz, 1967) then gives the change in the geoid level. The formula runs like this

$$N = a \int_{O}^{\pi} \frac{\Delta g}{\gamma} F(\theta)\, d\theta$$

Where N is the height of the geoid over the ellipsoid of reference, a is the radius of the Earth, Δg is the average gravity anomaly in the spherical distance Θ, γ is the theoretical gravity and $F(\Theta)$ is the so-called Stokes's function.

By the normal use of Stokes's formula to find the undulations of the geoid it is a tacit condition that no mass is placed outside the geoid. If that is the case the reduction of the gravity anomaly leads to different regularized geoids (co-geoids) dependent on the manner of reduction used. However, this problem does not occur here, all the mass-transformations being made at zero-level.

The said procedure was used by Kivioja (1967). He approximated the ice with a point-mass. That may seem rather crude, but a comparison with the classical calculations by Helmert (1884) and Woodward (1888) shows that the result is not too bad, the more so as we here limit ourselves to consider conditions at some distance from the ice—because we do not wish to introduce the complications from the local isostasy.

Fig. 1 shows the geoid-deformation N_1 (unit M) caused by a deglaciation of equivalent height 1 m versus the surface distance Θ from the center of the icemass.

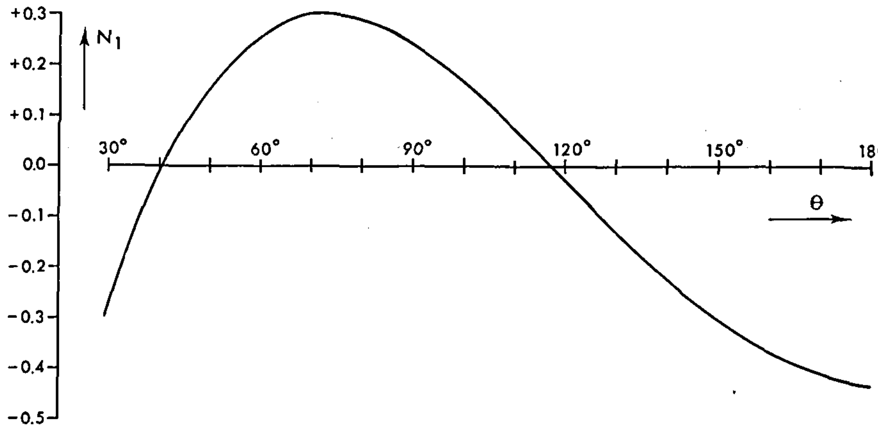

Fig. 1. Normalized geoid-deformation as a function of spherical distance. Vertical scale in metres.

A model of melting

The water-level will immediately follow the undulation mentioned above, the sea floor with a certain delay given by its relaxation time τ. Let us assume that an ice of equivalent height H begins to melt at the time $t = 0$. At a place in the distance Θ from the ice a geoid-anomaly x will begin to grow. We will try to find how x depends on t.

We will suppose that the ice melts with a constant velocity and that all the ice has melted at the time $t = T$. If the solid Earth did not react on the geoid-shift, we would have

(1) $$x(t) = \begin{cases} \frac{H \cdot N_1}{T} \cdot t, & 0 < t < T \\ H \cdot N_1 & t > T \end{cases}$$

where N_1 is the normalized geoid-deformation shown in fig. 1.

Let us first take the case $t<T$. The contribution to $\frac{dx}{dt}$ from the melting was $\frac{HN_1}{T}$ but this is counteracted by the moving of the sea-floor with the velocity $-\frac{x}{\tau}$. In all we then have

(2) $$\frac{dx}{dt} = \frac{HN_1}{T} - \frac{x}{\tau}$$

On condition that $x(0) = 0$, (2) has the solution

(3) $$x(t) = \frac{\tau}{T} \cdot H \cdot N_1 (1 - e^{-\frac{t}{\tau}}), \qquad 0 < t < T$$

When $t>T$ there is no growth, and we have only the delay. Our complete formula can then be written

(4) $$x(t) = \begin{cases} \frac{\tau}{T} H \cdot N_1 (1 - e^{-\frac{t}{\tau}}), & 0 < t < T \\ x(T) \cdot e^{-\frac{t-T}{\tau}}, & t > T \end{cases}$$

The formulas (1) and (4) are sketched in fig. 2.

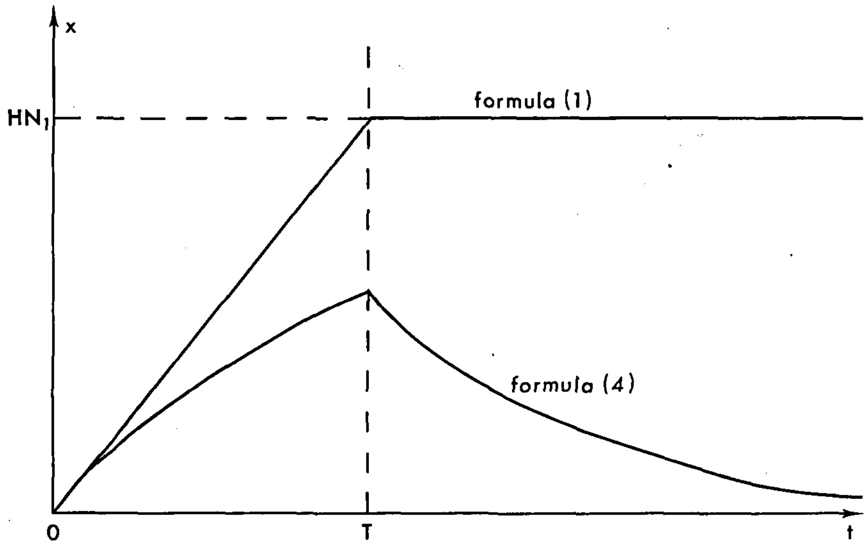

Fig. 2. Curves for the geoid-anomaly (x) as a function of time.

A trial insertion of numerical values

When inserting numerical values in formula (4) one is confronted with a lot of problems. Our present knowledge about the mass of the Pleistocene ice-masses (i. e. H) and their boundaries (i. e. N_1) is hardly precise enough. Also it is, of course, difficult to state when the melting started and stopped (in fact it is perhaps still in progress). And in all cases the melting has not been an uninterrupted linear process.

Another difficulty meets us in the choice of τ. Normally one fixes a relaxation time of about 4000 years or a little less for the isostatic upheaval in Fennoscandia and North America (Crittenden, 1967) while Gutenberg (1954) found about the double of that value. On the other hand, it is generally expected that a much longer relaxation time is to be found in connection with a shift of the Earth's axis of rotation.

We must also bear in mind that the application of formula (4) for one source or for more synchronous sources can never result in crossing anomaly-curves for the sea-level. However, if we presume two different sources to be active in different time-intervals, the phenomenon looked for may occur.

In a study of postglacial crustal movements Grachev & Dolokhanov (1969) state that "the deglaciation of Fennoscandia has begun about 13,000 and finished about 9,500 yrs. B. P. whereas in Canada the time limits of the deglaciation are 11,000 and 6,500 yrs. B. P. correspondingly." Let us try to use that estimate.

As to the division of the total amount of H—85 m—we use a table given by Daly (1934, p. 46). From that it follows that the two largest contributors to the sea-rise are North-America and Fennoscandia, approximately in the ratio 4:1 (See also Donn et al. 1962). Disregarding the smaller ice-masses we therefore set $H = 60$ m for the Wisconsin-ice and $H = 15$ m for the Würm-ice.

Let us pick up two eustatic curves of regional origin and from localities free of Pleistocene ice, viz. from the Christchurch area, New Zealand (Suggate, 1968) and from Bermuda (Neumann, 1969). The approximate distances and the corresponding values of N_1 are as follows: Bermuda-North America: 35°, ÷0.14. Bermuda-Europe: 55°, + 0,20. New Zealand-North America: 120°, 0.0 New Zealand-Europe: 160°, ÷ 0.36.

Having in mind that the whole calculation is tentative, we choose a simple τ value, viz. 3,300 (= 1/3 × 10,000) years. By the way, this choice is not critical.

Inserting the values mentioned above, we find the x-curves shown in the lower part of fig. 3. Note that these curves are not eustatic, but their de-

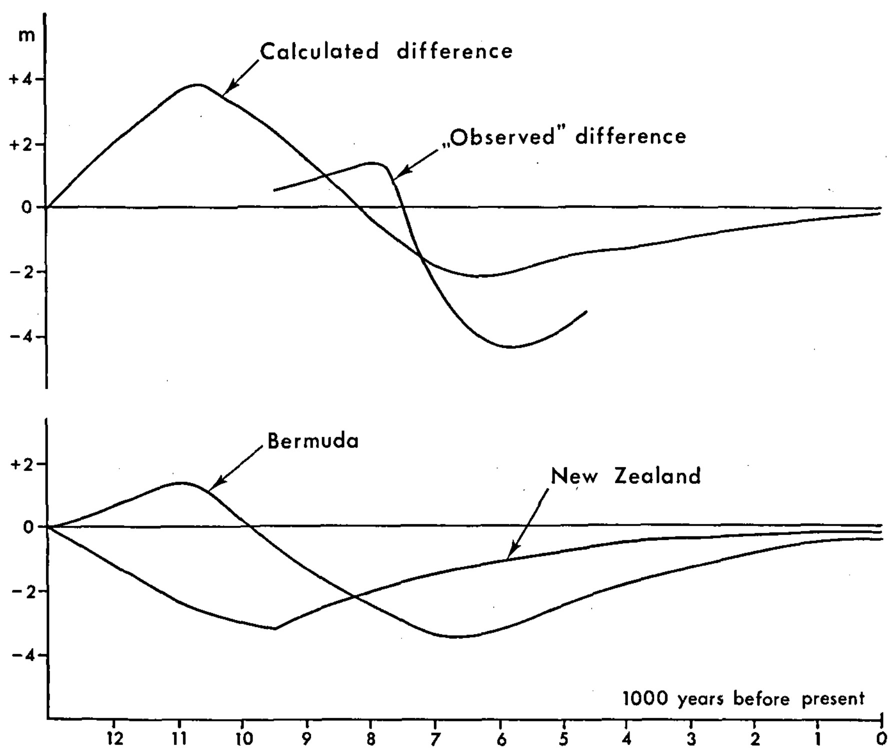

Fig. 3. Comparison of eustatic curves from Bermuda and Christchurch (New Zealand).

parture from the eustatic curve we would have found on an instantaneously yielding Earth.

In the upper part of fig. 3 the difference between the two x-curves is shown. For comparison the "observed" difference-curve taken from Mörner (1971) is also shown. As will be seen the order of magnitude for the differences is rather realistic including the sign. Remembering that we did not take into account that the rate of deglaciation must have been very uneven, we cannot expect a better fit. On the other hand, there are so many unknown parameters involved in the problem that the only safe conclusion must be that the possibility exists that the geoid-deformation causes some of the discrepancies.

Dansk sammendrag

En ofte overset virkning af en deglaciation er den geoide-deformation, som den giver anledning til. Havoverfladen indstiller sig øjeblikkeligt efter en ny geoide-form, men havbunden med en vis forsinkelse. Det medfører, at eustatiske kurver bestemt ud fra forskellige stabile områder ikke nødvendigvis er sammenfaldende. De parametre, der dirigerer mekanismen, er ikke særlig godt kendt. Under antagelsen af det russiske resultat, at Wisconsin- og Würm-istiderne ikke er helt samtidige, er der forsøgsvis foretaget en beregning af forskellen mellem de eustatiske kurver, der er bestemt fra Bermuda og fra New Zealand. Resultatet stemmer så nogenlunde med observationerne, og der drages den konklusion, at geoide-deformationen – i alt fald delvis – kan være årsag til de observerede uoverensstemmelser.

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